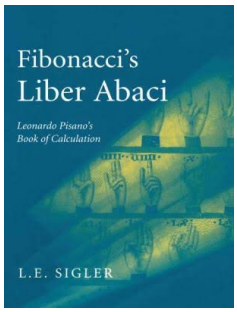
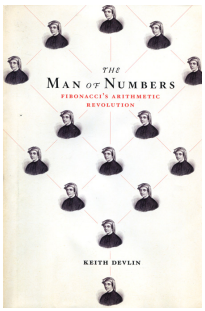
Fibonacci's Liber Abaci, Laurence Sigler, Sources and studies in the history of mathematical and physical sciences, Springer Verlag, (2002), ISBN:978-0-387-40737-1 (hbk). viii+633 p.
The man of numbers: Fibonacci's arithmetic revolution, *Keith Devlin*, Walker Books/Bloomsburry, (2011) ISBN 978-14088 2248 7 (hbk), 192 p.
Finding Fibonacci: The quest to rediscover the forgotten mathematical genius who changed the world, *Keith Devlin*, Princeton University Press, (2017) ISBN 978-06911 7486 0 (hbk), 256 p.






Keith Devlin

Ask random persons in the street if they know Fibonacci and there is a reasonable chance that they do. 'Didn't he have something about rabbits?' they may add. Some may know the Fibonacci numbers and their relation with the golden section. But much of the folk knowledge about Fibonacci is fake. Alternative facts so to speak.

To begin with his name. The man who lived from about 1170 till about 1250. Not much is known about him but according to the customs of those days, his name should be Leonardo Pisano (Leonard of Pisa), since we know that his home town was Pisa, where his father Guilielmo Bonacci was a well to do merchant. Fibonacci is a nickname invented by historian Guillaume Libri in 1838. It was inspired because in his (by now) most famous work *Liber abbaci*[1] (1202) he announced himself as *filius Bonacci* although his father's name was not Bonaccio. So instead of "son" he meant to say "of the Bonacci family".

His book *Liber abbaci* is a thick book of some 800 pages written in Latin, in which Fibonacci explains the Hindu-Arabic place-value numeral system with the nine digits and the zero. That system was already known a millennium earlier by the Indians and was used by the Arabs. Fibonacci learned about it when he joined his father in Bugia (currently known as Béjaïa in Algeria). He recognized the benefits of this system over the then usual Roman numeral system that was then used in Europe which required the use of finger arithmetic or of an instrument (the abacus[2]) and usually an accountant to do all the computations for the conversion of the sizes, weights, and currency for trading and for the emerging banking system. His book explains the system, how to compute with it and for its larger part it consists of practical examples of conversion, of computing interest and profit. One of the many examples is the rabbit example which gives rise to the so called Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34,.... That example was also known for a long time by the Indians in connection with Sanskrit prosody. It was popularized by the number theorist Édouard Lucas in the 19th century who called them Fibonacci numbers.


A page of *Liber Abbaci* with the Fibonacci sequence in the right margin

The recurrence relation $F_{n+1} = F_n + F_{n-1}$ for the Fibonacci numbers easily leads to the limit

[1] Fibonacci used this spelling with double b, although abaci is more common.
[2] *Liber abbaci* means 'Book of calculation'. Thus it is not explaining how to use the abacus, but on the contrary it explains how one can do without.

$\lim_{n\to\infty} F_{n+1}/F_n = \varphi = 1.618033988\ldots$, which by many is believed to appear in nature in many instances from the human body to phyllotaxis to the shape of seashells and that it should be the divine ratio[3] to be used in art creations and architecture. This story is debunked on mathematical grounds in Devlin's book.

So what is then the truth known about Fibonacci? We actually do not know much about the person. We only know his books[4]. But Fibonacci had a very keen marketing or outreach strategy. His *Liber abbaci* had many practical examples, but it was written in Latin, the language of the academics and the learned, not the language of the merchants and the bankers. However there exists hundreds of copies of *libri abbaco* which are vernacular light-versions of the *Liber abbaci*, and there must have been lessons to instruct the new system to the practitioners, so when this is added to the advantage it did indeed offer over the Roman numeral system, it took only few decades to have the new system generally accepted.

Typus Arithmeticae, is a woodcut from the book *Margarita Philosophica*, by Gregor Reisch, Freiburg, 1503. Boethius (left using the Hindu-Arab system) and Pythagoras (right using the abacus).

Thus if Fibonacci is known for his sequence of Fibonacci numbers illustrating the fertility of imaginary rabbits, then it is for the wrong reason. He is the one who introduced the numeral system to the Western world which is now used all over the planet. It is the only language that is understood worldwide. This is the reason for the outspoken admiration that Devlin has for Fibonacci. In his 2017 book be keeps repeating the tremendous importance of the invention of the Hindu-Arabic numeral system and how Fibonacci's way of spreading the news among the people for whom this really mattered was more important that al-Khwārizmī's *al-Jabr*.

Devlin's book *The man of numbers* from 2011 is the result of his exploration after he realized that no proper biography of Fibonacci existed and so little of the man was known. He is a mathematician, but after discovering his talent for writing popular mathematics books he reoriented his career and is very active as a writer, a blogger and appears often on television. He feels somehow related to Fibonacci after he learned how Fibonacci contributed to popularize the Hindu-Arabic system.

In fact, this is one of the highlights in *The man of numbers*. Fibonacci was forgotten for a number of centuries. Luca Pacioli in his *Summa de arithmetica*, (1494) refers to Fibonacci, which triggered Pietro Cossali in his history of mathematics book (1797) to point to Fibonacci as the originator of our numeral system. It was however as recent as 2003 when the missing link between the *Liber abbaci* and the vernacular *libri abbaco* was found. Fibonacci had referred several times to a *Liber minoris guise* (a Liber abbaci-light so to speak) or a *Libro di merchaanti*, that he had written, but it had never been found. Rafaella Franci eventually could identify a book from 1290 in the *Biblioteca Riccardiana* in Florence written in Umbria vernacular that directly refers to Fibonacci. This proved that he was not only the inspirator, but also the instigator of the European arithmetic revolution both in content and in form.

Of course, Devlin also explains whatever little bit is known about Fibonacci's life and of the time he lived in. Actually Fibonacci wrote several books and became rather famous in his time. He became a guest of Emperor Frederick II and in 1240, the Republic of Pisa granted him a salary for his services. The latter declaration is the last known historical trace of the man. Furthermore Devlin discusses the competing numeral systems and the Arabic sources that must have influenced Fibonacci. Naturally the contents of the *Liber abbaci* is extensively discussed, which he studied from

[3] A term introduced by Luca Pacioli in 1509. The name golden ratio is from Martin Ohm in 1835. But the number was known since antiquity.

[4] Although no original manuscript is left. We only know his revised edition from 1228. The probably oldest copy still available is from around 1275.

the first English translation by L. Sigler that appeared in 2002. Of course the basic operations, first for integers, then for numbers with fractional part. He starts with multiplication (the operation for which the new system is much better than the Roman numerals) and only then addition, subtraction, and division. The treatment follows strict logical rules in the tradition of Euclid's *Elements*. He introduces also the methods, old ones from the Arabic algebraic literature and new ones of his own invention. Casting out nines, rule of three, method of false position, and of double false position or *elchataym*[5],...It is richly illustrated with fully elaborate examples, and several chapters are devoted to practical problems (price of a product, investment, profit, money change, and metal alloys. The trailing extensive chapters are about computation with square and cubic roots and the method of *almuchabala*[6].

Statue of Fibonacci by Giovanni Paganucci, completed in 1863, in the Camposanto di Pisa.

Devlin concludes his book with a discussion of Fibonacci's legacy and influence which should not be the Fibonacci sequence not the golden section and *The Fibonacci Quarterly* journal is interesting for the applications, but there are no deep mathematics involved.

Devlin's most recent book is about his personal 'adventures' during the-making-of *The man of numbers*. It summarizes what is already in that book, but it is told from the viewpoint of the author. Much depended on good luck, like how he became a math expositor, how he visited Italy at the correct time, meeting the right people, and Sigler's translation published.

His quest has some funny aspects because of language communication problems, and the laidback Italian culture. When he is looking for a statue of Leonardo of Pisa and asks the tourist information center, they insist that he probably means Leonardo da Vinci. He also tells the story of the publication of Sigler's book. In fact Laurence Sigler succumbed to cancer in 1997 when the translation was finished up to editorial details. His wife Judith decided to do that but then there was the publisher who abandoned the project. The disks lost, it required a hacker to recover most of the text, from the computer but the typesetting was lost. When Springer was interested to publish the text they required a LaTeX version, which Judith had to learn, but finally against all odds, the book was published in 2002, exactly 800 year after Fibonacci had finished his original.

Devlin also describes his emotions when he is holding these old manuscripts that he, after some trouble, could finally leaf through, and there are two chapters of what came after the publication of *The man of numbers*. He draws a parallel between Fibonacci causing the arithmetic revolution and Steve Jobs who changed the way people thought about and handled computers with WIMPS[7]. In fact you can find a slide show called *Leo & Steve* about this on vimeo: vimeo.com/93390473 (Part 1) and vimeo.com/93532834 (Part 2) and another show with many pictures related to the *Man of Numbers* on www.maa.org/external_archive/devlin/Fibonacci.pdf. The last chapter is about a publication by William Goetzmann in which it is explained that Fibonacci describes in his *Liber abbaci* the basics of present-value analysis[8] and thus may be called the originator of modern finance.

Adhemar Bultheel

[5] This is equivalent to linear interpolation.

[6] This is the algebraic method of the *al-jabr* spelled out as *Al-kitāb al-mukhtaṣar fī ḥisāb al-ğabr wa'l-muqābala* which is *The compendious book on calculation by completion and balancing* in which a problem is solved by performing operations on an equation but keeping the 'balance' between the two sides.

[7] Windows, Icons, Menus, Pointers.

[8] This is a method for comparing the relative economic value of differing payment streams, taking into account the changing value of money over time. The present value of a euro is less than its future value because of its investment and interest potential.